

MA 8109 SUMMARY LEC 8

Int. theory $\rightarrow$ probab. theory $\rightarrow$ stoch. proc. $\rightarrow$ stoch. int. $\rightarrow$ SDEs $\rightarrow \dots$

Brownian motion: $B_0 = 0$, $\Delta B_k \sim N(0, \Delta t_k)$, $\Delta B_1, \dots, \Delta B_k$ independent

Dyadic rationals: $D := \bigcup_{n=1}^{\infty} D_n$, $D_n = \{0, \frac{1}{2^n}, \frac{2}{2^n}, \dots, 1\}$ dense in $[0,1]$

i.i.d $N(0,1)$ r.v.'s: $X_1, X_2, \dots$

Discrete B.M. on D :

$$D_0 : \tilde{B}(0) = 0, \quad \tilde{B}(1) = X_1$$

$$D_0 \rightarrow D_1 : \tilde{B}(0), \quad \tilde{B}(\frac{1}{2}) := \frac{1}{2}(\tilde{B}(0) + \tilde{B}(1)) + 2^{-1} \cdot X_2, \quad \tilde{B}(1)$$

$\vdots$

$$D_{m-1} \rightarrow D_m :$$

$$\underbrace{\tilde{B}(\frac{2k-1}{2^m})}_{\in D_m} := \frac{1}{2}(\underbrace{\tilde{B}(\frac{k-1}{2^{m-1}})}_{\in D_{m-1}} + \tilde{B}(\frac{k}{2^{m-1}})) + 2^{-\frac{m+1}{2}} X_i, \quad i = 2^{m-1}, \dots, 2^m - 1$$